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The Subset Size Sum Sequence: Structural Properties and Mathematical Applications

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ABSTRACT

The Subset Size Sum Sequence is defined as the sum of the cardinalities of all subsets of an n-element set. This study establishes its closed form $SC_n = n2^{n-1}$ and investigates its fundamental properties, including recurrence relations, generating functions, and relationships with known integer sequences. Furthermore, applications in combinatorics, graph theory, and discrete mathematics are presented. The findings demonstrate that the sequence provides a unified counting interpretation across various mathematical structures.

Keywords: *Subset Size Sum Sequence, Combinatorics, Discrete mathematics, Integer sequences, Generating functions, Graph theory*

INTRODUCTION

The Subset Size Sum Sequence is an integer sequence obtained by summing the cardinalities of all subsets of an n-element set. This sequence arises naturally in combinatorics through the study of power sets, subset enumeration, and counting arguments. Its simple closed form and rich structural behavior reveal meaningful connections with several well-known integer sequences and combinatorial objects. Because of these properties, the Subset Size Sum Sequence provides an important framework for understanding relationships among discrete structures, recurrence relations, and enumerative interpretations. This study investigates its definition, properties, relationships with other integer sequences, and applications in combinatorics and graph theory, thereby contributing to the broader field of discrete mathematics.

This study explores the Subset Size Sum Sequence in order to expand mathematical understanding of its structure, properties, and applications in combinatorics. Specifically, it aims to define the Subset Size Sum Sequence and establish its closed form. It further seeks to identify and prove its fundamental properties, including its recursive relation, second-order recurrence relation, explicit forms, summation identities, and generating function.

Moreover, the study aims to determine the relationships of the Subset Size Sum Sequence with other known integer sequences such as Woodall numbers, Cullen numbers, Eulerian numbers, Jacobsthal numbers, Mersenne numbers, and Qurra ibn Thabit numbers. Finally, it seeks to examine its applications in various combinatorial and graph-theoretic contexts, including the number of edges in the n-dimensional hypercube, nonattacking king arrangements on a chessboard, spanning trees of $K_{2,n}$, tatami tilings, compositions of integers, and the total number of variables appearing in product expansions. Through these objectives, the study highlights the significance of the Subset Size Sum Sequence in discrete mathematics.

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METHODS

The study employed the descriptive and expository methods of research. The descriptive method was used to systematically examine the structure, behavior, and relationships of the Subset Size Sum Sequence with other mathematical sequences and combinatorial objects. The expository method was applied to present the definitions, theorems, proofs, and derived formulas in a clear, logical, and organized manner.

Through these methods, the study developed a comprehensive discussion of the sequence, from its intuitive definition based on the power set of an n-element set to its formal algebraic and combinatorial interpretations. This approach enabled the study to explain the sequence rigorously while making its mathematical significance more accessible.

RESULTS AND DISCUSSION

1. Definition and Closed Form of the Subset Size Sum Sequence

1.1. Intuitive Definition of the Subset Size Sum Sequence

The study defined the Subset Size Sum Sequence, denoted by SC_n, as the sum of the cardinalities of all subsets of an n-element set.

1.2. Closed Form of the Subset Size Sum Sequence

The study established that the closed form of the Subset Size Sum Sequence is

$$SC_n = n \cdot 2^{n-1}.$$

2. Properties of the Subset Size Sum Sequence

2.1. Recursive Relation of the Subset Size Sum Sequence

The study established that the Subset Size Sum Sequence satisfies the recurrence relation

$$SC_n = 2SC_{n-1} + 2^{n-1},$$

with initial condition $SC_1 = 1$.

2.2. Second-Order Recurrence of the Subset Size Sum Sequence

The study established that the Subset Size Sum Sequence also satisfies the second-order recurrence

$$SC_n = 4SC_{n-1} - 4SC_{n-2},$$

with initial conditions $SC_0 = 0$and $SC_1 = 1$.

2.3. Closed Form for Even Indices of the Subset Size Sum Sequence

The study showed that for even indices, the Subset Size Sum Sequence is given by

$$SC_{2k} = k \cdot 4^k.$$

2.4. Binomial-Sum Representation of the Subset Size Sum Sequence

The study established that the Subset Size Sum Sequence may be expressed as

$$SC_n = \sum_{k=1}^{n} k \binom{n}{k}.$$

2.5. Factorial-Sum Form of the Subset Size Sum Sequence

The study further established the factorial-sum form of the sequence as

$$SC_n = n! \sum_{k=1}^{n} \frac{1}{(k-1)!\,(n-k)!}.$$

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2.6. Generating Function of the Subset Size Sum Sequence

The study established that the generating function of the Subset Size Sum Sequence is

$$G(x) = \frac{x}{(1-2x)^2}.$$

3. Relationships of the Subset Size Sum Sequence

3.1 Subset Size Sum Sequence and Woodall Numbers

Let SC_n denote the Subset Size Sum Sequence and let W_n denote the nth Woodall number. The study established that

$$SC_n = \frac{W_n + 1}{2}.$$

3.2 Subset Size Sum Sequence and Cullen Numbers

Let SC_n denote the Subset Size Sum Sequence and let C_n denote the nth Cullen number. The study established that

$$SC_n = \frac{C_n - 1}{2}.$$

3.3 Subset Size Sum Sequence and Eulerian Numbers

Let SC_n denote the Subset Size Sum Sequence and let $A(n,1)$ denote the Eulerian number counting permutations of n elements with exactly one ascent. The study established that

$$SC_n = \frac{nA(n,1) + n^2 + n}{2}.$$

3.4 Subset Size Sum Sequence and Jacobsthal Numbers

Let SC_n denote the Subset Size Sum Sequence and let $J_n$denote the nth Jacobsthal number. The study established that

$$SC_n = \frac{3n}{2}J_n + \frac{n}{2}(-1)^n.$$

3.5 Subset Size Sum Sequence and Mersenne Number

Let SC_n denote the Subset Size Sum Sequence and let M_n denote the nth Mersenne number. The study established that

$$SC_n = \frac{n}{2}(M_n + 1).$$

3.6 Subset Size Sum Sequence and Qurra ibn Thabit Numbers

Let SC_ndenote the Subset Size Sum Sequence and let $p_n$denote the nth Qurra ibn Thabit number. The study established that

$$SC_n = \frac{n(p_n + 1)}{3}.$$

4. Applications of Subset Size Sum Sequence

4.1. *Subset Size Sum Sequence and the Number of Edges of Q_n*

Let $\mid E(Q_n) \mid$ denote the number of edges of the n-dimensional hypercube Q_n. The study established that

$$\mid E(Q_n) \mid = SC_n = n2^{n-1}.$$

4.2. *Subset Size Sum Sequence and Nonattacking Kings on a $2 \times 2(n-1)$Chessboard*

The study established that the number of ways to place $n-1$nonattacking kings on a $2 \times 2(n-1)$chessboard is

$$SC_n = n2^{n-1}, n \geq 2.$$

4.3. *Subset Size Sum Sequence and Spanning Trees of $K_{2,n}$*

Let $T(K_{2,n})$ denote the number of spanning trees of the complete bipartite graph $K_{2,n}$. The study established that

$$T(K_{2,n}) = SC_n = n2^{n-1}.$$

4.4. Subset Size Sum Sequence and Tatami Tilings of an $n \times n$ Square with nMonomers

Let $T(n)$ denote the number of tatami tilings of an $n \times n$square with exactly $n$monomers. The study established that

$$T(n) = \begin{cases} SC_n = n2^{n-1}, & \text{if } n \text{ is even,} \\ 0, & \text{if } n \text{ is odd.} \end{cases}$$

4.5. Subset Size Sum Sequence and the Sum of All Parts in the Compositions of n

The study established that the total sum of all parts appearing in all compositions of n is

$$SC_n = n2^{n-1}.$$

4.6. Subset Size Sum Sequence and the Total Number of Variables in $(x_1 + 1)(x_2 + 1) \cdots (x_n + 1)$

The study established that the total number of variables appearing in all terms of the complete expansion of $(x_1 + 1)(x_2 + 1) \cdots (x_n + 1)$ is $SC_n = n2^{n-1}$.

CONCLUSION

The study of the Subset Size Sum Sequence revealed that it is a significant and versatile sequence in combinatorics and discrete mathematics. Its definition through the power set of an n-element set leads naturally to the closed form $SC_n = n2^{n-1}$, which serves as the basis for many of its important properties and applications. The established recurrence relations, summation forms, and generating function provide a deeper understanding of its structure and behavior.

Furthermore, the sequence was shown to have substantial relationships with several classical integer sequences, demonstrating its relevance in number theory. Its applications to graph theory, tilings, chessboard problems, compositions, and algebraic expansions further highlight its usefulness as a counting tool and mathematical model. Overall, the findings of the study contribute to the enrichment of combinatorial mathematics and may serve as a basis for further research on integer sequences and their applications.

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