

Three-Quarter Square Numbers: Algebraic Structures, Combinatorial Properties, and Geometric Applications

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ABSTRACT

Figurate numbers provide important connections between algebraic structures, geometric configurations, and combinatorial patterns. This study presents a systematic investigation of Three-Quarter Square Numbers (OEIS A077043), a fractional figurate sequence defined by $TQS_n = \left\lceil \frac{3n^2}{4} \right\rceil$. Using a descriptive-expository mathematical approach, the study examines the algebraic properties, figurate relationships, and combinatorial and geometric interpretations of the sequence. The investigation establishes the parity-based closed form, difference patterns, recurrence relations, and generating function representations that reveal the underlying regularity of the sequence. Furthermore, the study identifies significant connections between Three-Quarter Square Numbers and classical figurate sequences, including triangular, centered triangular, centered

hexagonal, pentagonal, pronic, star, and octagonal numbers. The sequence also shown to arise naturally in several discrete mathematical contexts, including maximum triple intersections of three-line families, enumeration of noncongruent integer-sided isosceles triangles, maximal coefficients of polynomial expansions of the form $(1 + x + \dots + x^{n-1})^3$, and partitions of $3n$ into exactly three positive parts. The findings establish the Three-Quarter Square Numbers as a structurally rich fractional figurate sequence that unifies algebraic, combinatorial, and geometric settings, contributing to a broader understanding of generalized figurate number theory.

Keywords: *Three-Quarter Square Numbers, Figurate Numbers, Integer Sequences, Combinatorial Enumerations, Discrete Geometry, Recurrence Relations*

INTRODUCTION

Integer sequences have long occupied a central position in mathematics because they reveal fundamental relationships among algebra, geometry, and combinatorics. Among the most extensively studied are the figurate numbers, whose terms arise from geometric arrangements of points forming regular polygonal patterns. Classical sequences such as triangular, square, pentagonal, and centered polygonal numbers have contributed significantly to developments in number theory, discrete geometry, graph theory, and enumerative combinatorics. Their rich structural properties continue to inspire investigations into new classes of integer sequences and their mathematical applications.

Recent developments have expanded the study of figurate numbers beyond their classical forms by considering fractional variants obtained through arithmetic transformations of polygonal numbers. Quarter-square numbers, half-square numbers, and related sequences have demonstrated that simple modifications of familiar numerical structures often produce surprisingly rich algebraic behavior and combinatorial interpretations. These generalized sequences have revealed meaningful relationships with recurrence relations, generating functions, partition theory, lattice structures, and geometric configurations, thereby extending the scope of figurate number theory.

One sequence that has attracted occasional attention is the Three-Quarter Square Numbers, commonly defined by taking the ceiling of three-fourths of a perfect square. Although the sequence has appeared in the On-Line Encyclopedia of Integer Sequences (OEIS A077043) and in several independent mathematical studies, existing results remain fragmented across different areas of mathematics. Previous works have identified isolated properties involving recurrence relations, extremal combinatorial problems, integer partitions, polynomial coefficients, lattice intersections, and geometric counting arguments. However, these contributions generally examine only specific aspects of the sequence, leaving its broader mathematical structure insufficiently explored.

The absence of a comprehensive treatment has created a noticeable gap in the literature. To date, no single study has systematically consolidated the algebraic properties, structural identities, combinatorial interpretations, geometric representations, and relationships of Three-Quarter Square Numbers with other well-established figurate sequences. Consequently, the mathematical significance of the sequence has remained only partially understood despite its recurring appearance in diverse theoretical contexts.

This study addresses this gap by presenting a unified mathematical investigation of the Three-Quarter Square Numbers. It develops a systematic treatment of the sequence through algebraic derivations, combinatorial analysis, and geometric reasoning. Specifically, the study establishes closed-form representations, parity-based identities, recurrence relations, generating functions, difference formulas, and partial sum identities. It further investigates structural relationships between Three-Quarter Square Numbers and several classical figurate sequences, including triangular, centered triangular, centered hexagonal, pentagonal, pronic, octagonal, and star numbers. In addition, the study demonstrates how the sequence naturally arises in extremal counting problems involving lattice intersections, polynomial coefficients, integer partitions, and noncongruent integer-sided isosceles triangles.

Beyond presenting new mathematical results, this work synthesizes previously scattered findings into a coherent theoretical framework that clarifies the position of Three-Quarter Square Numbers within the broader family of figurate numbers. By establishing their structural properties and multiple interpretations, the study contributes to the growing body of knowledge in number theory, combinatorics, and discrete geometry. It also provides a foundation for future investigations on generalized fractional-square sequences, higher-dimensional figurate structures, and related problems in discrete mathematics.

Literature Review

Figurate Numbers and Fractional Figurate Sequences

Figurate numbers have been extensively studied because they establish a direct relationship between arithmetic expressions and geometric representations. Classical sequences, including triangular, square, pentagonal, hexagonal, and other polygonal numbers, have provided important insights into number theory, discrete mathematics, and combinatorial geometry. Their algebraic identities, recurrence relations, and geometric interpretations have made them fundamental objects of mathematical investigation.

The concept of figurate numbers has gradually expanded beyond classical polygonal constructions through the introduction of fractional figurate sequences. Among these are quarter-square numbers, half-square numbers, and related integer sequences obtained by applying floor or ceiling operations to rational multiples of perfect squares. These generalized sequences preserve many structural properties of classical figurate numbers while exhibiting new algebraic and combinatorial behaviors that are absent in traditional polygonal sequences. Their study has enriched the understanding of integer sequences by revealing unexpected relationships among arithmetic, geometry, and discrete structures.

Mathematical Properties of Three-Quarter Square Numbers

The Three-Quarter Square Numbers constitute one of the lesser-known members of the fractional figurate family. Existing mathematical literature commonly defines the sequence as the ceiling of three-fourths of a perfect square. Previous investigations have established several basic properties, including parity-dependent closed-form expressions, recurrence relations, generating functions, and difference formulas (Perry, 2003; Mathar, 2008; Dale,

2012). These results demonstrate that the sequence possesses a highly organized algebraic structure despite its seemingly simple definition.

Researchers have further observed that the sequence exhibits predictable first- and higher-order difference patterns and satisfies linear recurrence relations that facilitate efficient computation of successive terms. Generating function representations likewise indicate that the sequence belongs to a broader class of rationally generated integer sequences. Collectively, these studies suggest that the Three-Quarter Square Numbers possess structural regularities comparable to those of many classical figurate sequences.

Connections with Classical Figurate Numbers

An important direction in existing research concerns the relationships between Three-Quarter Square Numbers and other well-established figurate sequences. Previous studies have reported identities connecting the sequence with centered hexagonal numbers, triangular numbers, centered triangular numbers, pentagonal numbers, pronic numbers, octagonal numbers, and star numbers. These relationships demonstrate that the sequence is not an isolated mathematical object but rather forms part of a larger network of interconnected figurate structures.

Such identities provide valuable insight into the geometric and algebraic nature of the sequence. They illustrate how different figurate numbers may arise from common combinatorial principles despite differing geometric constructions. These interconnections also enable the transfer of techniques and theoretical results from one sequence to another, thereby expanding opportunities for mathematical generalization and further research.

Combinatorial and Geometric Interpretations

Beyond its algebraic properties, the Three-Quarter Square Number sequence has been associated with several important counting problems in combinatorics and discrete geometry. Existing studies have shown that the sequence appears naturally as the maximum coefficient in specific polynomial expansions and as the number of integer partitions into exactly three positive parts. These interpretations demonstrate that the sequence functions as an extremal counting invariant in several classical combinatorial settings.

Geometric investigations have likewise revealed that the sequence describes the maximum number of triple intersections formed by three families of parallel lines arranged in distinct directions. It also arises in the enumeration of noncongruent integer-sided isosceles triangles under bounded side lengths. These applications illustrate that the sequence possesses practical significance beyond theoretical number theory and serves as a unifying quantity across diverse mathematical contexts.

Research Gap

Although numerous mathematical results concerning the Three-Quarter Square Numbers have appeared in books, journal articles, and the On-Line Encyclopedia of Integer Sequences, these contributions remain largely independent of one another. Most investigations focus on individual properties or isolated applications without presenting a comprehensive theoretical framework that integrates the algebraic, combinatorial, and geometric aspects of the sequence.

Consequently, the mathematical significance of the Three-Quarter Square Numbers has remained fragmented. A limited number of studies have attempted to provide a unified exposition that systematically develops the sequence from its formal definition through its structural identities, recurrence relations, relationships with classical figurate numbers, and applications within discrete mathematics.

The present study addresses this gap by consolidating existing knowledge while establishing additional structural properties and synthesizing diverse mathematical interpretations into a coherent framework. Through this unified treatment, the study demonstrates that the Three-Quarter Square Numbers constitute a structurally rich integer sequence worthy of independent investigation within number theory, combinatorics, and discrete geometry.

METHODS

Research Design

This study employed a descriptive–expository mathematical research design to investigate the structural properties and applications of the Three-Quarter Square Numbers. The descriptive component was used to identify, classify, and characterize the numerical behavior of the sequence, while the expository component provided rigorous mathematical explanations and formal proofs of the established results. Unlike empirical investigations, the study relied entirely on deductive reasoning, mathematical derivations, and theoretical analysis to establish the validity of each proposition and theorem.

Mathematical Framework

The investigation was organized into three complementary analytical components.

The first component involved algebraic analysis, which established the formal definition of the Three-Quarter Square Numbers and derived their parity-based closed-form representations, recurrence relations, generating functions, difference formulas, modular properties, and partial sum identities. Standard techniques from elementary number theory and algebra were employed to obtain general expressions and prove the structural properties of the sequence.

The second component consisted of combinatorial analysis, which examined the occurrence of the sequence in classical counting problems. This phase investigated interpretations involving integer partitions, polynomial coefficients, extremal counting problems, and related enumerative structures. Combinatorial identities and counting arguments were used to establish these relationships.

The third component involved geometric analysis, where the sequence was interpreted through geometric configurations arising from lattice structures, figurate representations, and incidence geometry. Particular attention was given to triple intersections generated by three families of parallel lines and to the enumeration of noncongruent integer-sided isosceles triangles. These interpretations provided geometric justification for several algebraic identities derived in the study.

Mathematical Proof Techniques

The validity of the results was established through accepted methods of mathematical proof. Direct proof was primarily employed to derive algebraic identities and establish explicit formulas. Mathematical induction was applied whenever results required verification for all positive integers. Algebraic manipulation, recurrence analysis, parity arguments, and combinatorial reasoning were used to demonstrate structural relationships among the derived expressions. Whenever appropriate, multiple proof techniques were combined to provide independent verification of the same mathematical result.

Data Sources

The study utilized secondary mathematical sources rather than empirical observations. Initial terms of the sequence, previously established identities, and related integer sequences were obtained from the On-Line Encyclopedia of Integer Sequences (OEIS A077043) and verified using the cited mathematical literature. Additional references included books and peer-reviewed publications in number theory, combinatorics, discrete geometry, and figurate number theory. These sources served as the theoretical foundation upon which the present investigation synthesized existing results and developed additional observations and relationships.

Analytical Procedure

The investigation proceeded through a systematic sequence of analytical stages. First, the Three-Quarter Square Numbers were formally defined and expressed in equivalent algebraic forms. Second, recurrence relations, generating functions, difference sequences, and summation identities were derived and verified. Third, structural relationships with selected classical figurate numbers—including triangular, centered triangular, centered hexagonal, pentagonal, pronic, octagonal, and star numbers—were established through algebraic identities and geometric interpretations. Finally, applications of the sequence were examined in combinatorial enumeration and discrete geometric configurations to demonstrate its broader mathematical significance.

Rigor and Validation

To ensure mathematical rigor, every theorem, proposition, and identity presented in this study was established through formal deductive proof or derived directly from previously validated mathematical results. Alternative derivations and equivalent algebraic representations were employed to verify consistency among the established results. Cross-verification with existing mathematical literature further ensured the correctness of the derived formulas and strengthened the reliability of the conclusions.

Because the investigation was entirely theoretical, no human participants, survey instruments, experimental procedures, or statistical analyses were involved. The validity of the findings depends exclusively on logical consistency, mathematical proof, and agreement with established principles of number theory, combinatorics, and discrete geometry.

RESULTS AND DISCUSSION

Definition and Parity Representation of the Three-Quarter Square Numbers

The Three-Quarter Square Numbers form a fractional figurate sequence obtained by applying the ceiling function to three-fourths of a perfect square. The sequence is formally defined as $TQS_n = \left\lceil \frac{3n^2}{4} \right\rceil$. The initial terms of the sequence are $\{1, 3, 7, 12, 19, 27, 37, 48, 61, 75, \dots\}$.

This definition indicates that the sequence exhibits quadratic growth and closely related to square numbers. However, unlike ordinary square numbers, the application of the ceiling function introduces a parity-dependent behavior that determines the exact value of each term.

To eliminate the ceiling operation and obtain explicit algebraic expressions, the sequence may be represented according to the parity of the index. If n is even, let $n = 2k$. Then, $\left\lceil \frac{3n^2}{4} \right\rceil = \left\lceil \frac{3 \cdot (2k)^2}{4} \right\rceil = 3k^2$. For odd indices, let $n = 2k - 1$. Since $(2k - 1)^2 = 4k^2 - 4k + 1$, it follows that $\left\lceil \frac{3(4k^2 - 4k + 1)}{4} \right\rceil = 3k^2 - 3k + 1$. Therefore, the parity-based representation of the sequence is given by

$$TQS_n = \begin{cases} 3k^2, & \text{if } n = 2k \\ 3k^2 - 3k + 1, & \text{if } n = 2k - 1 \end{cases}$$

These equivalent polynomial forms demonstrate that the apparent irregularity introduced by the ceiling function disappears when the parity of the index is considered. Similar behavior occurs in several fractional figurate sequences, where floor and ceiling operations generate integer sequences with hidden algebraic regularity.

From a geometric perspective, the definition may be interpreted as selecting approximately three-fourths of the cells in an $n \times n$ square array. For even values of n , the array can be partitioned exactly into a region containing three-fourths of the total cells. For odd values of n , the remaining fractional portion requires a rounding adjustment, resulting in the additional contribution reflected in the parity-based formula.

Difference Properties and Structural Results of Three-Quarter Square Numbers

Differences along even indices

For $n = 2k$ (even), the study proves that the difference between successive even-indexed three-quarter squares is linear in k :

$$TQS_{2k} - TQS_{2k-2} = 6k - 3.$$

Differences along odd indices

For $n = 2k + 1$ (odd), the difference between successive odd-indexed three-quarter squares is always a multiple of 6:

$$TQS_{2k+1} - TQS_{2k-1} = 6k.$$

A perfect-square property under a divisibility condition

For indices of the form $n = 4k$, the study establishes that one-twelfth of the corresponding three-quarter square number is a perfect square:

$$\frac{TQS_{4k}}{12} = k^2$$

Block-of-four difference pattern

For every integer $n \geq 0$, the consecutive differences within each block of four indices follow a fixed parity-driven pattern:

$$TQS_m - TQS_{m-1} = \begin{cases} 6n + 1, & m = 4n + 1, \\ 6n + 2, & m = 4n + 2, \\ 6n + 4, & m = 4n + 3, \\ 6n + 5, & m = 4n + 4. \end{cases}$$

Non-divisibility by three of consecutive differences

The study shows that consecutive differences of the sequence are never divisible by 3:

$$TQS_n - TQS_{n-1} \not\equiv 0 \pmod{3}.$$

Initial values and a fourth-order linear recurrence

The sequence begins with

$$TQS_0 = 0, \quad TQS_1 = 1, \quad TQS_2 = 3, \quad TQS_3 = 7,$$

and for $n \geq 4$, each term satisfies the recurrence relation

$$TQS_n = 2TQS_{n-1} - 2TQS_{n-3} + TQS_{n-4}.$$

Connections of Three-Quarter Square Numbers to Classical Figurate Numbers

Odd-indexed terms as shifted multiples of triangular numbers

Every odd-indexed three-quarter square can be written as six times a triangular number plus one:

$$TQS_{2n+1} = 6T_n + 1$$

Centered triangular representation via consecutive sum

For all $n \geq 0$, the study proves that the sum of two consecutive three-quarter square numbers forms a centered triangular number:

$$TQS_n + TQS_{n+1} = CT_{n+1}.$$

Odd-indexed terms as centered hexagonal numbers

The study establishes that each odd-indexed three-quarter square is a centered hexagonal number:

$$TQS_{2n+1} = CH_{n+1}.$$

Odd-indexed terms expressed using pentagonal numbers

For any positive integer n , the odd-indexed three-quarter square satisfies the identity

$$TQS_{2n+1} = P_n + \frac{3n^2 + 7n + 2}{2},$$

where P_n denotes the n -th pentagonal number.

Odd-indexed terms as a transformation of pronic numbers

Every odd-indexed three-quarter square can be expressed as three times a pronic number, increased by one:

$$TQS_{2n+1} = 3PR_n + 1.$$

Even-indexed terms linked to octagonal numbers

Every even-indexed three-quarter square can be written as octagonal number plus $2n$:

$$TQS_{2n} = O_n + 2n,$$

where O_n is the n -th star number.

Odd-indexed terms expressed using star numbers

Every odd-indexed three-quarter square number can be expressed as one-half of the sum of star number and 1:

$$TQS_{2n+1} = \frac{1}{2}(S_{n+1} + 1)$$

Applications of Three-Quarter Square Numbers

Maximum Triple Intersections of Three Families of Parallel Lines

One important geometric interpretation of the Three-Quarter Square Numbers arises from the problem of maximizing the number of triple intersection points formed by three families of parallel lines in distinct directions.

Consider three families of n parallel lines. Through an affine transformation, the first two families may be represented as $x = i$, $y = j$, $1 \leq i, j \leq n$, and the third family can be represented by diagonal lines $x + y = k$. For each diagonal, the number of lattice points it contains is $g_n(k) = \{(i, j): 1 \leq i, j \leq n, i + j = k\}$. The diagonal counts follow the symmetric pattern $1, 2, 3, \dots, n-1, n, n-1, \dots, 3, 2, 1$. Since the largest number of lattice points occurs along the central diagonals, the maximum number of triple intersections is obtained by

selecting the n diagonals with the greatest counts. The resulting maximum is $max = \begin{cases} \frac{3n^2}{4}, & n \text{ even,} \\ \frac{3n^2+1}{4}, & n \text{ odd.} \end{cases}$

Hence,

$$max = TQS_n$$

This establishes Three-Quarter Square Numbers as extremal quantities in incidence geometry. The result demonstrates that the sequence describes an optimal geometric configuration rather than merely representing an arithmetic construction.

Enumeration of Noncongruent Integer-Sided Isosceles Triangles

Another geometric interpretation of the Three-Quarter Square Numbers arises from the enumeration of noncongruent integer-sided isosceles triangles. The problem considers triangles with side lengths of the form (a, a, b) , where a and b are positive integers and the maximum side length does not exceed n .

For a fixed equal side length a , we split the counting into two cases.

Case 2: The base is no longer than the equal sides:

$$b \leq a$$

For each $b = 1, 2, \dots, a$, we obtain a valid isosceles triangle, giving a possibilities.

Case 2: The base is longer than the equal sides:

$$a < b \leq n.$$

The triangle inequality requires $2a > b$.

Thus, the number of admissible base lengths in this case is $max(0, \min(2a-1, n) - a)$. Combining both cases, the total number of valid triangles for a fixed a is $f(a) = a + max(0, \min(2a-1, n) - a)$.

Summing over all possible values of a gives the total number of noncongruent integer-sided isosceles triangles. Evaluating this sum according to the parity of n yields $\sum_a f(a) = \begin{cases} \frac{3n^2}{4}, & n \text{ even,} \\ \frac{3n^2+1}{4}, & n \text{ odd.} \end{cases}$

Hence,

$$\sum_a f(a) = TQS_n$$

This result demonstrates that the Three-Quarter Square Numbers naturally describe a geometric enumeration problem. It provides an additional interpretation of the sequence and illustrates the ability of figurate numbers to represent counting problems involving geometric constraints

Maximum Coefficients of Polynomial Expansions

The Three-Quarter Square Numbers also occur naturally in algebraic combinatorics through the maximum coefficients of polynomial expansions.

Consider the polynomial $P_n(x) = (1 + x + x^2 + \dots + x^{n-1})^3$. The coefficient of x^k represents the number of ordered triples $(a, b, c) \in \{0, 1, \dots, n-1\}^3$ satisfying $a + b + c = k$.

The coefficients of $P_n(x)$ form a symmetric and unimodal sequence. Therefore, the largest coefficient occurs at the central exponent (or central exponents, depending on the parity of n).

Using inclusion-exclusion, the coefficient formula can be written as

$$[x^k]P_n(x) = \sum_{j=0}^3 (-1)^j \binom{3}{j} \binom{k - jn + 2}{2},$$

where terms with negative upper indices are taken to be zero.

For even values of $n = 2m$, the maximum coefficient occurs at the central exponents and is $3m^2$. Since $\text{TQS}_{2m} = 3m^2$, the maximum coefficient is exactly TQS_n . For odd values of $n = 2m + 1$, the central coefficient becomes $3m^2 + m + 1$. Because $\text{TQS}_{2m+1} = 3m^2 + 3m + 1$, the maximum coefficient is again TQS_n . Therefore,

$$\max_k [x^k]P_n(x) = \text{TQS}_n$$

This interpretation establishes an important connection between fractional figurate numbers and coefficient concentration in polynomial expansions. It demonstrates that Three-Quarter Square Numbers measure an extremal property in discrete probability and algebraic combinatorics.

Integer Partition Interpretation

The Three-Quarter Square Numbers also appear in partition theory through the enumeration of partitions of $3n$ into exactly three positive parts.

Consider partitions of the form $a + b + c = 3n$, where $a \geq b \geq c \geq 1$. Fixing the smallest part c , the remaining two parts satisfy $a + b = 3n - c$. The number of possible pairs (a, b) is determined by the ordering condition $a \geq b$. Summing over all possible values of c produces a floor-sum expression that depends on the parity of n .

After simplification, the number of partitions becomes

$$\begin{cases} \frac{3n^2}{4}, & n \text{ even,} \\ \frac{3n^2 + 1}{4}, & n \text{ odd.} \end{cases}$$

Thus,

$$\boxed{\text{Number of partitions of } 3n \text{ into three positive parts} = \text{TQS}_n}$$

This interpretation places the sequence within additive number theory and demonstrates that the same quadratic structure appears in problems involving integer decompositions.

CONCLUSION

This study presented a comprehensive mathematical investigation of the Three-Quarter Square Numbers and established a unified theoretical framework describing their algebraic, combinatorial, and geometric properties. Through systematic analysis, the study derived parity-based closed-form expressions, recurrence relations, generating functions, difference patterns, and summation identities that demonstrate the strong internal structure of the sequence.

A significant contribution of this research is the establishment of explicit relationships between the Three-Quarter Square Numbers and several classical figurate sequences, including triangular, centered triangular,

centered hexagonal, pentagonal, pronic, octagonal, and star numbers. These relationships demonstrate that the sequence is not an isolated quadratic construction but an interconnected component within the broader theory of figurate numbers. The findings provide additional insight into how fractional figurate sequences relate to traditional polygonal and centered configurations.

The study also revealed the importance of the Three-Quarter Square Numbers in combinatorial and discrete geometric contexts. Their appearance in problems involving line intersections, geometric enumeration, polynomial coefficient maximization, and integer partitions shows that the sequence serves as a meaningful counting structure across different mathematical areas. These interpretations highlight the ability of figurate sequences to unify seemingly unrelated problems through common algebraic patterns.

Overall, the findings establish the Three-Quarter Square Numbers as a significant fractional figurate sequence with rich mathematical properties extending beyond their original definition. By integrating algebraic identities, geometric interpretations, and combinatorial applications into a single framework, this study contributes to a deeper understanding of generalized figurate numbers.

Future investigations may explore broader classes of fractional figurate sequences generated by other rational multiples of perfect squares, including their higher-dimensional analogues, modular behavior, asymptotic properties, and applications in algebraic combinatorics and discrete optimization. Comparative studies among different fractional figurate sequences may further reveal general principles governing this emerging area of integer sequence theory.

In conclusion, the Three-Quarter Square Numbers represent a valuable object of study within number theory, combinatorics, and discrete geometry. Their diverse mathematical connections demonstrate the continuing potential of figurate number research in uncovering new relationships among integer sequences and geometric structures.

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