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Twin Edge Coloring of Mycielski Graph of Some Families of Graph

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ABSTRACT

Twin edge coloring is a proper edge coloring in which the vertex coloring induced by the sum of the colors assigned to incident edges is also proper. The research aimed to define and construct the Mycielski graph of a given graph, examine its fundamental structural properties, and determine the twin chromatic index of the Mycielski graphs of paths, cycles, stars, and tadpole graphs. Using a mathematical and expository research approach, the study applied established concepts and results in graph theory to analyze the structural characteristics and edge-coloring behavior of these graph families under the Mycielski construction. The findings revealed important properties of Mycielski graphs, including their order, size, degree, radius, diameter, girth, and circumference, and established results on the twin edge coloring and twin-chromatic indices of the selected graph families. The analysis demonstrated that the structural modifications introduced by the Mycielski construction significantly affect twin edge coloring behavior. Consequently, the study concluded that Mycielski graphs provide a valuable framework for investigating twin edge coloring and related graph coloring parameters. The results advance twin edge coloring theory and lay foundation for future research on Mycielskian graphs and other graph coloring problems.

Keywords: *Twin edge coloring, Twin chromatic index, Mycielski graph, and graph invariants.*

INTRODUCTION

Graph theory is one of the major branches of discrete mathematics that studies graphs, mathematical structures used to represent relationships among objects. A graph consists of vertices and edges, where vertices represent objects and edges represent the connections between them. As noted by N. Biggs, E. K. Lloyd, and R. J. Wilson, *"The origins of graph theory are humble, even frivolous."* Despite its modest beginnings, graph theory has developed into an important field with numerous applications in computer science, engineering, transportation, communication systems, scheduling, and optimization, thanks to its ability to model networks and relational structures.

Among the many topics in graph theory, graph coloring is one of the most extensively studied. In general, graph coloring involves assigning colors or labels to the elements of a graph subject to specific restrictions to distinguish adjacent or related elements, thereby avoiding conflicts and enabling the analysis of structural properties. One important type of graph coloring is edge coloring, in which colors are assigned to edges so that no two adjacent edges share the same color. Over time, more specialized coloring concepts have emerged, including those that combine edge colorings with induced vertex conditions.

One such concept is twin edge coloring, which is defined as a proper edge coloring of a graph in which the induced vertex coloring, obtained from the sum of the colors assigned to the incident edges of each vertex, is

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also proper. The minimum number of colors required for such a coloring is called the twin chromatic index. This concept is significant because it integrates edge labeling and vertex distinction within a single framework, making it a more restrictive and mathematically rich form of graph coloring. Consequently, determining the twin chromatic index of graphs and graph families has become an important problem in theoretical graph theory.

Another concept closely related to this study is the Mycielski graph construction, a well-known graph transformation that generates a new graph from an existing one while increasing its chromatic complexity without introducing triangles in certain cases. Owing to its distinctive structural properties, the Mycielski graph has become an important object of study, particularly in investigations involving graph coloring parameters and extremal graph theory. The interaction between twin edge coloring and the Mycielski graph construction provides a meaningful and worthwhile area for mathematical analysis, motivating further investigation into the twin chromatic index of Mycielskian graphs.

METHODS

This study employed descriptive and expository research methods. The descriptive method was used to present and explain the essential concepts, definitions, and properties related to the Mycielski graph and twin edge coloring. Specifically, it described the structure of the Mycielski graph of a given graph, including its fundamental properties such as order, size, and maximum degree. This method was also used to present the graph families examined in the study and the corresponding results for their twin chromatic indices.

The expository method was employed to develop and discuss the study's theoretical framework. Through this approach, the researcher systematically presented the mathematical definitions, theorems, proofs, and graph constructions related to the twin edge coloring of the Mycielski graph of selected graph families. It also provided a logical and coherent explanation of the results' derivation, demonstrating how the conclusions were established using existing mathematical concepts, proven results, and deductive reasoning.

RESULTS AND DISCUSSION

Definition of Mycielski Graph

A Mycielski graph (*also known as Mycielskian*) of a graph, denoted by $\mu(G)$, consists of every vertex $v_i \in G$ connected to its duplicated vertex u_i such that every u_i is adjacent to all vertices in $N_G(v_i)$ and all u_i is connected to a new vertex w.

Consider the Path Graph P_5 in Figure 1,

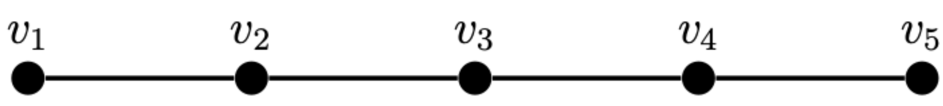

Figure 1. The *Path Graph* P_5.

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following the definition, the resulting Mycielskian of P_5 is shown in Figure 2 below.

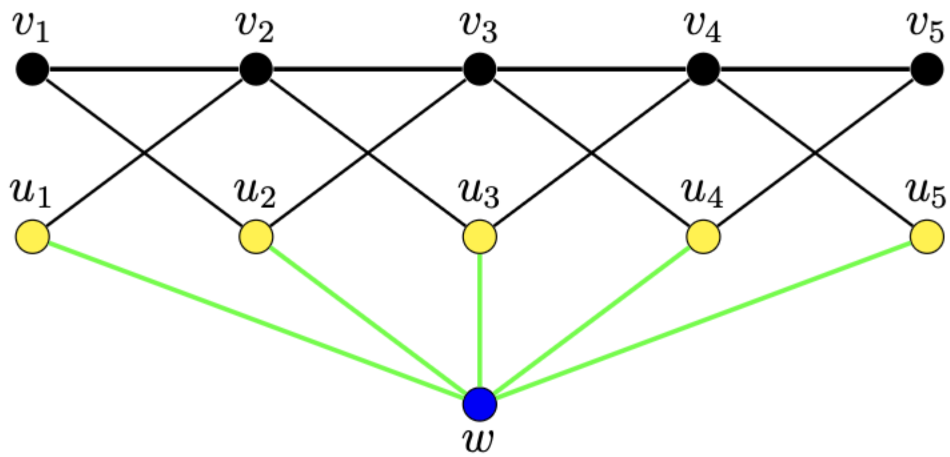

Figure 2. The Mycielskian $\mu(P_5)$.

Properties of Mycielski Graph

General Properties of Mycielski Graph

Let G be a graph of order n, then

$$|\mu(G)| = 2n + 1.$$

Let G be a graph of order n and size m, then

$$|E(\mu(G))| = 3m + n.$$

Let G be a graph of order n, then

$$\Delta(\mu(G)) = max\{2\Delta(G), n\}.$$

If a graph does not contain any 3-cycle subgraphs, then its Mycielskian is triangle-free.

If a graph G is triangle-free, then

$$\omega\big(\mu(G)\big) = max\{\omega(G), 2\}.$$

For any graph G,

$$\chi\big(\mu(G)\big) = \chi(G) + 1.$$

Graph Invariants of Mycielskian of Some Families of Graph

Let P_n be a Path Graph of order n, for $n \geq 2$, then

$$\delta\big(\mu(P_n)\big) = 2.$$

$$rad\big(\mu(P_n)\big) = 2.$$

$$circ\big(\mu(P_n)\big) = 2n.$$

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$$g\big(\mu(P_n)\big) = \begin{cases} 5 & if\ n = 2 \\ 4 & if\ n \geq 3 \end{cases}.$$

$$diam\big(\mu(P_n)\big) = \begin{cases} 2 & if\ n = 2,3 \\ 3 & if\ n = 4 \\ 4 & if\ n \geq 5 \end{cases}.$$

Let C_n be a Cycle Graph of order n, for $n \geq 4$, then

$$\delta\big(\mu(C_n)\big) = 3.$$

$$rad\big(\mu(C_n)\big) = 2.$$

$$circ\big(\mu(C_n)\big) = 2n + 1.$$

$$g\big(\mu(C_n)\big) = 4.$$

$$diam\big(\mu(C_n)\big) = \begin{cases} 2 & if\ n = 4,5 \\ 3 & if\ n = 6,7 \\ 4 & if\ n \geq 8 \end{cases}.$$

Let S_n be a Star Graph of order n, for $n \geq 2$, then

$$\delta\big(\mu(S_n)\big) = 2.$$

$$rad\big(\mu(S_n)\big) = 2.$$

$$circ\big(\mu(S_n)\big) = 5.$$

$$g\big(\mu(S_n)\big) = \begin{cases} 5 & if\ n = 2 \\ 4 & if\ n \geq 3 \end{cases}.$$

$$diam\big(\mu(S_n)\big) = 2.$$

Twin Chromatic Index of Some Families of Graph

Let P_n be a Path Graph of order n, for $n \geq 2$, then

$$\grave{\chi_t}(\mu(P_n)) = \begin{cases} 4 & if\ n = 3 \\ 5 & if\ n = 2,4 \\ n & if\ n \geq 5 \end{cases}.$$

Let C_n be a Cycle Graph of order n, for $n \geq 3$, then

$$\grave{\chi_t}(\mu(C_n)) = \begin{cases} 5 & if\ n = 3,4 \\ n & if\ n \geq 5 \end{cases}.$$

Let S_n be a Star Graph of order n, for $n \geq 3$, then

$$\grave{\chi_t}(\mu(S_n)) = 2n - 2.$$

CONCLUSION

The Mycielski graph of a graph G is a graph transformation that constructs a larger graph $\mu(G)$ by adding a shadow vertex for every original vertex of G, and a new root vertex w. This construction preserves the original graph while introducing new adjacency relations between shadow vertices and the open neighborhoods of their corresponding original vertices, as well as between the root vertex and all shadow vertices. Hence, the Mycielski graph provides a useful structure for investigating how graph transformations affect coloring parameters.

The twin edge coloring of Mycielski graphs depends strongly on the structure of the original graph family. For path graphs, cycle graphs, and star graphs, the study showed that the twin chromatic index is influenced by the maximum degree and the induced vertex sums resulting from the edge coloring. In particular, the Mycielski construction imposes additional restrictions: the edge coloring must be proper, and the induced vertex coloring must also be proper. Thus, the results of this study provide specific twin-edge coloring values, bounds, and constructions for the selected families of graphs.

References

Abedin, P., Akbari, S., Demange, M., & Ekim, T. (2017). Complexity of the improper twin edge coloring of graphs. *Graphs and Combinatorics, 33*(4), 595–615. https://doi.org/10.1007/s00373-017-1782-7

Abramowitz, M., & Stegun, I. A. (Eds.). (1964). *Handbook of mathematical functions with formulas, graphs, and mathematical tables* (Applied Mathematics Series No. 55). National Bureau of Standards. https://doi.org/10.6028/NBS.AMS.55

Anantharaman, S. (2020). Twin edge coloring of total graph and graphs with twin chromatic index Δ + 2. *Applications and Applied Mathematics: An International Journal, 15*(1), Article 18.

Andrews, E., Helenius, L., Johnston, D., VerWys, J., & Zhang, P. (2014). On twin edge colorings of graphs. *Discussiones Mathematicae Graph Theory, 34*(3), 613–627. https://doi.org/10.7151/dmgt.1756

Bondy, J. A., & Murty, U. S. R. (2008). *Graph theory*. Springer.

Chartrand, G., & Zhang, P. (2009). *Chromatic graph theory*. Chapman & Hall/CRC. https://doi.org/10.1201/9781584888017

Chvátal, V. (1974). The minimality of the Mycielski graph. In R. A. Bari & F. Harary (Eds.), *Graphs and combinatorics*(Lecture Notes in Mathematics, Vol. 406, pp. 243–246). Springer. https://doi.org/10.1007/BFb0066446

Diestel, R. (2017). *Graph theory* (5th ed.). Springer. https://doi.org/10.1007/978-3-662-53622-3

Došlić, T. (2005). Mycielskians and matchings. *Discussiones Mathematicae Graph Theory, 25*(3), 261–266. https://doi.org/10.7151/dmgt.1279

Fisher, D. C., McKenna, P. A., & Boyer, E. D. (1998). Hamiltonicity, diameter, domination, packing, and biclique partitions of Mycielski's graphs. *Discrete Applied Mathematics, 84*(1–3), 93–105. https://doi.org/10.1016/S0166-218X(97)00126-1

Harary, F. (1969). *Graph theory*. Addison-Wesley.

Jensen, T. R., & Toft, B. (1995). *Graph coloring problems*. Wiley-Interscience.

Lin, W., Wu, J., Lam, P. C. B., & Gu, G. (2006). Several parameters of generalized Mycielskians. *Discrete Applied Mathematics, 154*(8), 1173–1182. https://doi.org/10.1016/j.dam.2005.11.001

Mycielski, J. (1955). Sur le coloriage des graphes. *Colloquium Mathematicae, 3*(2), 161–162. https://doi.org/10.4064/cm-3-2-161-162

Rajarajachozhan, R., & Sampathkumar, R. (2016). Twin edge colorings of certain square graphs and product graphs. *Electronic Journal of Graph Theory and Applications, 4*(1), 79–93. https://doi.org/10.5614/ejgta.2016.4.1.7

Tolentino, J. D. (2019). *On the twin chromatic indices of trees and some graphs with maximum degree 3* (Doctoral dissertation, Ateneo de Manila University).

Tolentino, J. D., Marcelo, R. M., & Tolentino, M. A. C. (2020). Twin chromatic indices of some graphs with maximum degree 3. *Journal of Physics: Conference Series, 1538*(1), Article 012004. https://doi.org/10.1088/1742-6596/1538/1/012004

Tolentino, J. D. L., Marcelo, R. M., & Tolentino, M. A. C. (2022). On twin edge colorings in *m*-ary trees. *Electronic Journal of Graph Theory and Applications, 10*(1), 131–149. https://doi.org/10.5614/ejgta.2022.10.1.8

West, D. B. (2001). *Introduction to graph theory* (2nd ed.). Prentice Hall.

Yap, H. P. (1996). *Total colourings of graphs* (Lecture Notes in Mathematics, Vol. 1623). Springer. https://doi.org/10.1007/BFb0092895