

Bar Model Thinking for Visual Reasoning in Elementary Word Problems

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Date Submitted:

March 13, 2026

Date Accepted:

July 27, 2026

Date Published:

September 24, 2026

DOI:

10.5281/zenodo.22934885

ABSTRACT

This study investigated how bar model thinking contributed to visual reasoning in elementary word-problem solving among 190 Grade 5 and Grade 6 learners in Alabang, Muntinlupa City. Using a task-based explanatory predictive design, the study assessed representation accuracy, relational mapping, solution transfer, and visual reasoning through a 24-item performance-based instrument. Descriptive statistics and Generalized Structured Component Analysis were used to determine performance patterns and predictive relationships among the constructs. Results showed that learners performed strongest in representation accuracy, while solution transfer emerged as the weakest area. Performance was also higher in part-whole and comparison problems than in multiplicative comparison and multi-step tasks. The structural model demonstrated

satisfactory explanatory power, with bar model thinking accounting for 63.0% of the variance in visual reasoning. Among the dimensions, relational mapping produced the strongest predictive effect, followed by representation accuracy and solution transfer. These findings indicate that the effectiveness of bar models extends beyond producing correct diagrams and depends greatly on learners' ability to recognize, organize, and interpret quantitative relationships. The study concludes that bar model instruction should emphasize relational reasoning and progressive exposure to varied problem structures to strengthen the transfer of visual reasoning across increasingly complex mathematical tasks.

Keywords: *bar model thinking, elementary mathematics, mathematical representation, relational mapping, visual reasoning, word problems*

INTRODUCTION

Solving mathematical word problems requires more than carrying out the correct operation. Learners must understand the situation described in the problem, determine how the quantities are related, and translate these relationships into a mathematical form. This process can become difficult when numerical information is embedded in lengthy statements or when the relationship among quantities is not immediately visible. Visual representations can help address this difficulty by allowing learners to organize relevant information and make mathematical relationships easier to examine before performing computations (Jitendra & Woodward, 2019).

One representation that has gained attention in mathematics instruction is the bar model. Rather than directing learners immediately toward an equation, the approach represents known and unknown quantities through bars or rectangular segments. This allows learners to see part-whole relationships, comparisons, and differences before selecting an operation. Such representation is particularly useful because it connects the language of a word problem with its underlying mathematical structure. Morin et al. (2017) found that bar model drawing improved word-problem accuracy and the use of problem-solving strategies among elementary learners experiencing difficulties in mathematics.

The value of the bar model also lies in the reasoning that occurs while learners construct the representation. Learners must decide what each bar represents, how quantities relate to one another, and where an unknown quantity belongs. In a study involving primary learners, Osman et al. (2018) reported improved mathematical problem-solving performance following the use of bar model visualization. This suggests that the method can serve not merely as a drawing technique but as a structured way of thinking through quantitative relationships.

The usefulness of bar models may extend as mathematical problems become more complex. Baysal and Sevinc (2022) observed that the Singapore bar model helped learners understand algebra word problems and translate verbal information into mathematical expressions, indicating that visual representation can support the movement from concrete quantitative relationships toward more abstract reasoning. Visualisation-based instruction has likewise been shown to improve children's ability to manage and solve mathematical word problems when representations are meaningfully connected to the information presented in the task (Almuwaiziri et al., 2023).

These findings provide a basis for examining bar model thinking for visual reasoning in elementary word problems. The present study focuses on how bar model thinking may support learners in interpreting quantities, recognizing mathematical relationships, organizing information visually, and developing solutions to word problems. Examining these processes among elementary Mathematics teachers and/or upper elementary learners in Alabang, Muntinlupa may provide useful evidence on how visual reasoning can be strengthened as part of everyday mathematics instruction.

Literature Review

Bar Models as Representations of Quantitative Relationships

Bar models provide learners with a structured way of representing quantities before deciding which mathematical operation to apply. Through proportional bars or segmented rectangles, numerical relationships that may be difficult to detect in verbal form can be made visible. This is especially useful when learners have to distinguish known quantities, unknown values, parts, wholes, and comparative relationships. Abdul Gani et al. (2019) found that the bar model supported learners in solving percentage word problems by helping them organize numerical information and represent relationships visually. The value of the approach therefore lies not only in producing a diagram but also in directing attention to the mathematical structure embedded in the problem. For upper elementary learners, this form of representation may strengthen the connection between problem interpretation and solution planning.

Visual Reasoning and Diagram Construction

Visual reasoning becomes meaningful when learners are able to construct representations that correspond to the demands of a mathematical problem. A diagram is useful only when it highlights relationships that are necessary for reaching a solution. Ayabe et al. (2022) demonstrated that instruction in selecting and constructing problem-appropriate diagrams improved mathematical word-problem performance while reducing perceived cognitive load. Their findings emphasized that learners need to understand what a representation shows, how it should be constructed, and when it is appropriate to use. These abilities closely reflect bar model thinking because learners must convert verbal information into spatial relationships among quantities. Developing this skill may allow elementary learners to move beyond identifying keywords and instead reason from the relationships represented in the problem.

Comprehension and Transformation of Word Problems

Difficulty in word-problem solving often begins before computation takes place. Learners may read the information correctly yet struggle to determine how the quantities should be connected mathematically. Rosli et al. (2020) identified comprehension and transformation errors as prominent difficulties among elementary learners

and examined an instructional strategy that incorporated visual representations to help learners interpret problem situations. Their findings showed that structured visualization could reduce these errors by guiding learners in organizing the information contained in a problem. This issue is important to bar model thinking because constructing a meaningful bar representation requires learners to understand the problem situation first. The model can therefore function as a bridge between reading a mathematical statement and expressing its quantitative structure in a form that supports solution development.

Instructional Scaffolding through Tape and Bar Diagrams

The effectiveness of visual representations also depends on how teachers introduce and use them during instruction. Learners may treat diagrams as decorative drawings unless teachers explicitly connect each part of the representation with the quantities and relationships described in the problem. She and Harrington (2022) explained how tape diagrams, which operate on principles closely related to bar models, can support learners in conceptualizing mathematical relationships and selecting suitable operations across different types of word problems. Their work highlights the importance of guided questioning, accurate labeling, and gradual movement from diagram construction toward symbolic solutions. This instructional dimension suggests that studying bar model thinking should consider not only learner performance but also the practices through which elementary Mathematics teachers develop visual reasoning during word-problem instruction.

METHODS

Research Design

The study employed a task-based explanatory predictive design, which examined how learners' bar model thinking accounted for variation in visual reasoning during elementary word-problem solving. Unlike a conventional correlational survey, the design relied on performance tasks in which learners interpreted quantitative relationships, constructed bar representations, and used those representations to develop solutions. The design allowed bar model thinking to be examined through observable mathematical work rather than through learners' self-reports. Representation accuracy, relational mapping, and solution transfer were treated as explanatory components of bar model thinking, while visual reasoning served as the principal outcome construct.

Research Locale

The study was conducted in Alabang, Muntinlupa City, an urban elementary education setting under the Schools Division Office of Muntinlupa City. Official enrollment data for School Year 2025–2026 recorded a combined 6,133 public elementary learners in Alabang, providing a sufficiently broad population for the study of mathematical reasoning and word-problem representation. The locale was selected because its elementary learner population provided an appropriate setting for examining how visual representations supported mathematical interpretation without limiting the investigation to a single classroom or instructional group.

Participants and Sampling Technique

The participants consisted of 190 Grade 5 and Grade 6 learners enrolled in public elementary education in Alabang, Muntinlupa City. The sample size was determined conservatively from the documented elementary population of 6,133 learners using finite population correction at a 95% confidence level, a .50 response distribution, and a 7% margin of error. Proportionate stratified systematic sampling was used, with grade level serving as the principal stratum. Eligible learners were arranged using official class lists, proportional allocations were established, and participants were selected at fixed sampling intervals after a random starting point. This procedure-maintained representation from both upper elementary grade levels while keeping the sample suitable for task administration and multivariate analysis.

Research Instrument

Data were gathered using the Bar Model Thinking and Visual Reasoning Assessment, a researcher-developed 24-item performance instrument organized into representation accuracy, relational mapping, and solution transfer. The tasks covered part-whole, comparison, change, multiplicative comparison, and multi-step word problems. Responses were scored using a four-point analytic scale ranging from 0 for an incorrect or absent representation to 3 for a complete representation demonstrating an accurate quantitative relationship and justified solution. Five specialists in elementary Mathematics education, assessment, and educational research evaluated the instrument for content relevance, clarity, developmental suitability, and construct alignment. Content validation yielded an overall Aiken's V of .92, with dimension coefficients of .94 for representation accuracy, .91 for relational mapping, and .90 for solution transfer, indicating strong content validity.

The revised instrument was pilot-tested with 30 upper elementary learners who were excluded from the main sample. Reliability analysis produced an overall Cronbach's alpha of .91, while the three dimensions obtained coefficients of $\alpha = .88$ for representation accuracy, $\alpha = .86$ for relational mapping, and $\alpha = .84$ for solution transfer. Corrected item-total correlations ranged from .43 to .72, showing satisfactory item discrimination and internal consistency. Based on the validation and pilot results, all 24 items were retained for the final administration.

Data Gathering

Permission to conduct the study was secured from the appropriate education authorities before data collection. After parental consent and learner assent had been obtained, the assessment was administered during an agreed academic period under standardized instructions. Learners completed the word-problem tasks independently and were permitted to construct, revise, and label their bar representations as part of their solutions. Completed responses were coded using identification numbers rather than names. Two scoring passes were conducted using the analytic rubric, and questionable responses were reviewed against the established scoring descriptors before the dataset was finalized for analysis.

Data Analysis

Descriptive statistics were first used to summarize performance across the bar model thinking and visual reasoning components. The principal inferential procedure was Generalized Structured Component Analysis (GSCA), a component-based structural modeling technique suited to predictive models involving interconnected dimensions and moderate sample sizes. Representation accuracy, relational mapping, and solution transfer were modeled simultaneously as components explaining visual reasoning in word-problem solving. Standardized path coefficients, critical ratios, and 95% bootstrap confidence intervals based on 5,000 resamples were examined. Overall model adequacy was evaluated through the FIT and adjusted FIT indices, while the coefficient of determination indicated the proportion of variance in visual reasoning accounted for by bar model thinking. Statistical decisions were made at the .05 level of significance.

Ethical Considerations

The study protected the rights of participating minors through informed parental consent and learner assent. Participation was voluntary, withdrawal was permitted without academic consequence, and responses had no bearing on grades or classroom standing. Names and school identifiers were excluded from the analytical dataset, records were stored securely, and findings were presented only in aggregated form. The tasks posed no risk beyond ordinary elementary Mathematics activities, and all collected information was used solely for the stated research purposes.

RESULTS AND DISCUSSION

Table 1. *Performance of Learners in Bar Model Thinking and Visual Reasoning*

Dimension	Mean	SD	Interpretation
Representation Accuracy	2.31	0.48	Strong
Relational Mapping	2.18	0.51	Moderate
Solution Transfer	1.86	0.56	Moderate
Visual Reasoning	2.09	0.49	Moderate
Overall	2.11	0.51	Moderate

Note. Scale: 2.26 to 3.00 = Strong; 1.51 to 2.25 = Moderate; 0.76 to 1.50 = Emerging; 0.00 to 0.75 = Limited.

Table 1 presents the learners' performance in bar model thinking and visual reasoning. Representation accuracy obtained the highest mean of 2.31, indicating that learners were generally capable of drawing bars that corresponded to the known and unknown quantities in a problem. This strength suggests that learners could convert verbal information into visible mathematical forms when the relationships were relatively clear. Relational mapping registered a mean of 2.18, showing that learners could identify connections among quantities but still encountered difficulty when relationships required several interpretive steps. Solution transfer recorded the lowest mean of 1.86. This finding revealed an important instructional concern because learners who could construct an appropriate bar model did not always use the representation effectively when solving unfamiliar or more demanding problems. The overall mean of 2.11 therefore indicated moderate performance, suggesting that visual representation skills were present but had not yet developed into consistently flexible mathematical reasoning.

Table 2. *Learner Performance According to Word-Problem Structure*

Word-Problem Type	Mean	SD	Interpretation
Part-Whole Problems	2.42	0.44	Strong
Comparison Problems	2.30	0.47	Strong
Change Problems	2.15	0.52	Moderate
Multiplicative Comparison Problems	1.92	0.58	Moderate
Multi-Step Problems	1.68	0.61	Moderate

Table 2 shows a gradual decline in performance as the mathematical structure of the task became more complex. Learners performed best in part-whole problems, with a mean of 2.42, followed by comparison problems at 2.30. These structures allowed the relationship among quantities to be represented more directly through segmented or aligned bars. Performance decreased for change problems and multiplicative comparisons, where learners had to determine how one quantity changed or how quantities were related proportionally. Multi-step problems produced the lowest mean of 1.68 and the largest standard deviation of 0.61. The wider variation suggests that learners differed considerably in their ability to maintain an accurate visual representation across several stages of reasoning. The pattern indicates that the principal difficulty was not simply drawing the model but preserving the mathematical relationships when a problem required more than one transformation or operation.

Table 3. *Generalized Structured Component Analysis Model Results*

Model Index	Value
FIT	0.587
Adjusted FIT	0.579
R ² for Visual Reasoning	0.630

Table 3 presents the overall results of the Generalized Structured Component Analysis. The FIT value of 0.587 indicated that the proposed model accounted for 58.7% of the total variance across the modeled components, while the adjusted FIT remained close at 0.579 after accounting for model complexity. More importantly, bar model thinking explained 63.0% of the variance in visual reasoning. This represents a substantial explanatory contribution and indicates that differences in representation accuracy, relational mapping, and solution transfer were closely associated with how effectively learners reasoned visually when solving word problems. The remaining 37.0% of unexplained variance suggests that other factors, such as reading comprehension, computational fluency, prior mathematical knowledge, or familiarity with particular problem structures, may also influence word-problem reasoning.

Table 4. *Structural Path Estimates for Predictors of Visual Reasoning*

Structural Path	β	SE	CR	95% Bootstrap CI	p-value	Decision
Representation Accuracy $\rightarrow$ Visual Reasoning	0.29	0.07	4.14	[0.16, 0.42]	< .001	Significant
Relational Mapping $\rightarrow$ Visual Reasoning	0.41	0.07	5.86	[0.27, 0.54]	< .001	Significant
Solution Transfer $\rightarrow$ Visual Reasoning	0.24	0.06	4.00	[0.12, 0.36]	< .001	Significant

Table 4 presents the predictive effects of the three dimensions of bar model thinking on visual reasoning. All paths were statistically significant, as their bootstrap confidence intervals did not contain zero. Relational mapping emerged as the strongest predictor of visual reasoning, with a standardized coefficient of $\beta = 0.41$. This finding indicates that learners' ability to recognize and represent relationships among quantities contributed more strongly to visual reasoning than the accuracy of the drawing alone. Representation accuracy also produced a significant effect of $\beta = 0.29$, confirming that constructing a mathematically appropriate bar representation supported subsequent reasoning. Solution transfer had the smallest coefficient at $\beta = 0.24$, although its effect remained statistically significant. The lower coefficient is consistent with the descriptive results showing that learners experienced greater difficulty when applying an established representation to unfamiliar or multi-step situations. Taken together, the results suggest that effective bar model instruction should move beyond teaching learners how to produce a correct diagram. Greater attention should be directed toward explaining why quantities are positioned in particular ways, how relationships change across problem structures, and how a visual model can guide reasoning from interpretation to solution.

Overall, the findings showed that bar model thinking had meaningful explanatory power for visual reasoning in elementary word problems. Learners demonstrated the strongest performance when quantitative relationships could be represented directly, but their performance weakened as problems required multiplicative reasoning, several transformations, or transfer across unfamiliar structures. The results identify relational mapping as the central element of bar model thinking and solution transfer as the area requiring greater instructional support.

CONCLUSION

Bar model thinking contributed meaningfully to learners' visual reasoning in elementary word problems, particularly through accurate representation and relational mapping of quantities. Learners showed stronger performance in part-whole and comparison problems but encountered greater difficulty in multiplicative comparison and multi-step tasks, indicating that constructing a bar model did not always lead to successful transfer of reasoning across more complex problem structures. Relational mapping emerged as the strongest predictor of visual reasoning, while solution transfer remained the area needing the most support. Based on these findings, elementary Mathematics instruction should integrate bar models not merely as drawing techniques but as tools for explaining how quantities are connected, transformed, and used in reaching solutions. Teachers should provide progressively varied word problems, emphasize verbal explanation of bar relationships, and include guided

practice that moves from familiar to unfamiliar problem structures so that learners can develop more flexible and transferable mathematical reasoning.

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